On Locality-Sensitive Orderings & Their Applications

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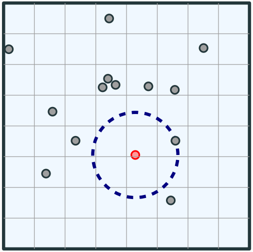
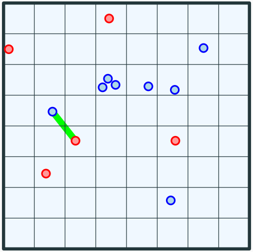
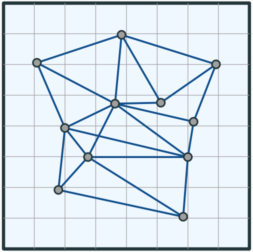
Low dimension proximity problems: $d = O(1)$



Nearest neighbor
[Indyk, Motwani '98]
[Liao *et al.* '01]
[Chan '02]



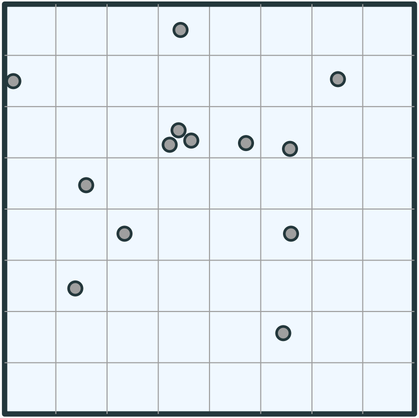
Closest pair problems
[Eppstein '95]
$\implies \approx O(\log^3 n)$



Spanners/MST
[Roditty '12]
[Gottlieb, Roditty '08]

Goal: Design dynamic data structures which return a $(1+\varepsilon)$-approximation

Ordering of points



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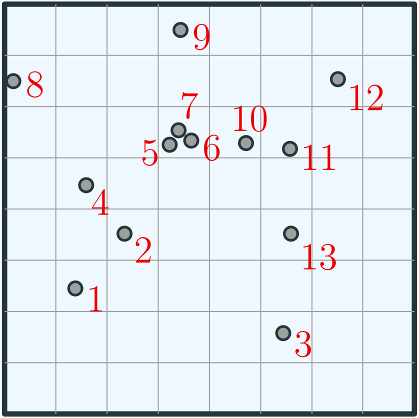
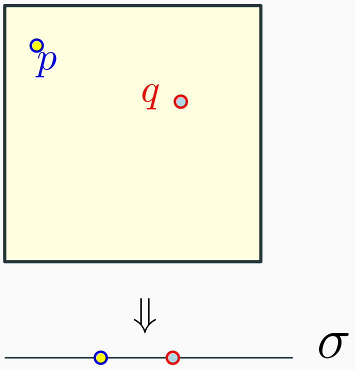


Locality-sensitive orderings



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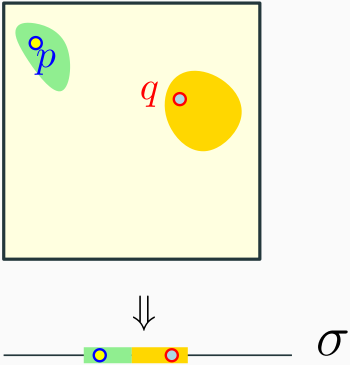

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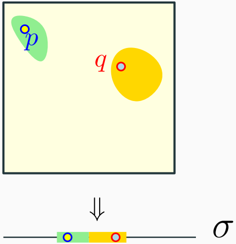

Definition: Locality-Sensitive Orderings

Let $\varepsilon \in (0, 1)$. A collection of orderings Π over $[0, 1)^d$ s.t. for all $p, q \in [0, 1)^d$, there exists a $\sigma \in \Pi$ where:

$$\forall p \prec_\sigma z \prec_\sigma q : \min(\|z - p\|, \|z - q\|) \leqslant \varepsilon \|p - q\|.$$

Main Theorem

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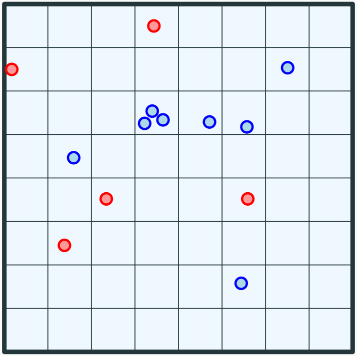
Theorem

There are locality-sensitive orderings of size $O\big((1/\varepsilon^d) \log(1/\varepsilon)\big)$.

Application: Bichromatic closest pair

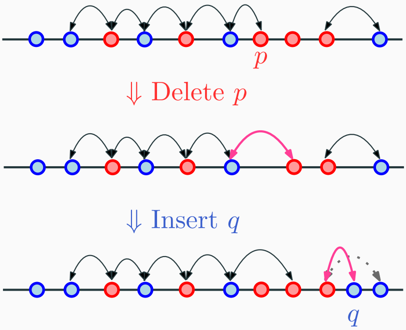


Problem

Maintain a pair (r', b') s.t. $\|r' - b'\| \leqslant (1 + \varepsilon) \cdot \min_{(r,b)} \|r - b\|$.

Application: Bichromatic closest pair

- Idea: Solve the 1D problem



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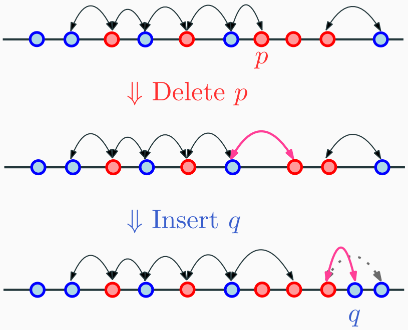
- Idea: Solve the 1D problem
- Maintain order in a binary tree



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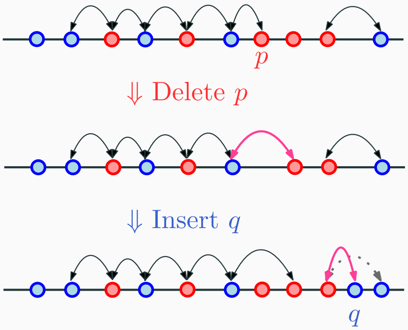
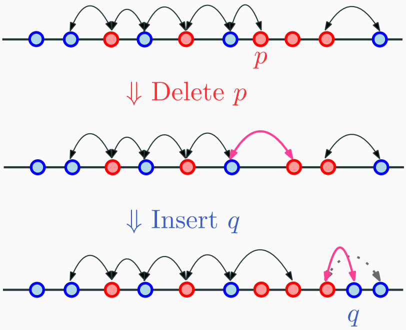
- Maintain min-heap of consecutive red/blue pairs



Application: Bichromatic closest pair

- Idea: Solve the 1D problem
- Maintain order in a binary tree
- Maintain min-heap of consecutive red/blue pairs
- Easily made dynamic



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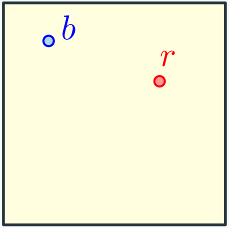

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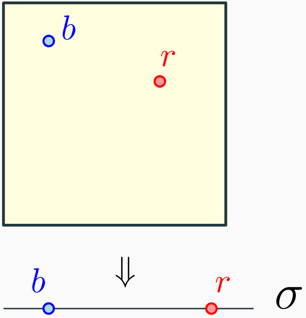

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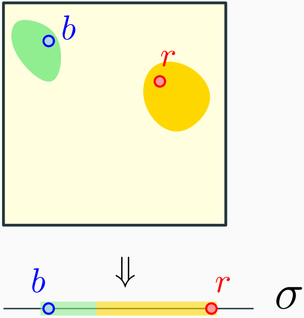

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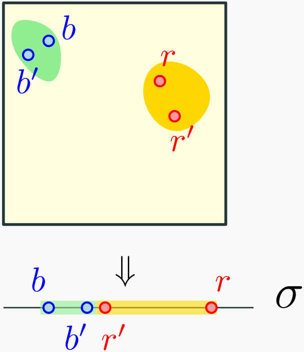

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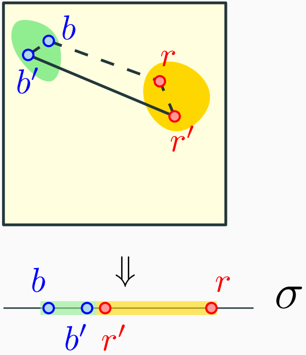

$$\|r' - b'\| \leqslant \|r' - r\| + \|r - b\| + \|b - b'\|$$
$$\leqslant (1 + 2\varepsilon)\|r - b\|$$

Applications

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4. Dynamic vertex-fault-tolerant spanners (new)
5. Approximate nearest neighbor (not new, [Chan '02])

References i

Timothy M. Chan.
Closest-point problems simplified on the RAM.
In *Proc. 13th ACM-SIAM Sympos. Discrete Alg.* (SODA), pages 472–473. SIAM, 2002.

T.-H. Hubert Chan, Mingfei Li, Li Ning, and Shay Solomon.
New doubling spanners: Better and simpler.
SIAM J. Comput., 44(1):37–53, 2015.

David Eppstein.
Dynamic Euclidean minimum spanning trees and extrema of binary functions.
Discrete Comput. Geom., 13:111–122, 1995.

Lee-Ad Gottlieb and Liam Roditty.
An optimal dynamic spanner for doubling metric spaces.
In *Proc. 16th Annu. Euro. Sympos. Alg.* (ESA), pages 478–489, 2008.

Piotr Indyk and Rajeev Motwani.
Approximate nearest neighbors: Towards removing the curse of dimensionality.
In *Proc. 30th ACM Sympos. Theory Comput.* (STOC), pages 604–613, 1998.

Sanjiv Kapoor and Xiang-Yang Li.
Efficient construction of spanners in d-dimensions.
CoRR, abs/1303.7217, 2013.

Swanwa Liao, Mario A. López, and Scott T. Leutenegger.
High dimensional similarity search with space filling curves.
In *Proc. 17th Int. Conf. on Data Eng.* (ICDE), pages 615–622, 2001.

Liam Roditty.
Fully dynamic geometric spanners.
Algorithmica, 62(3-4):1073–1087, 2012.